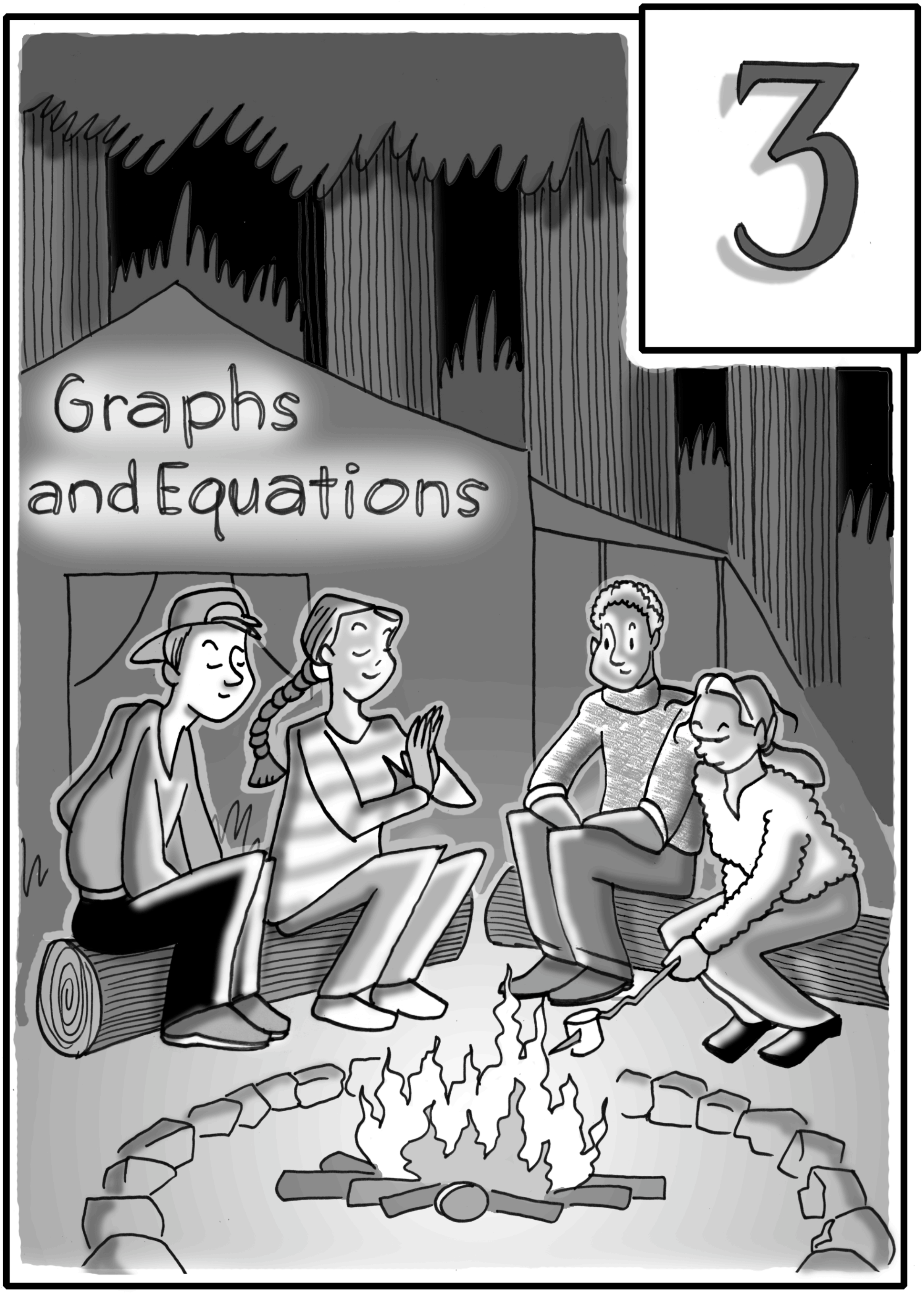

3
Graphs and Equations

CHAPTER 3 Graphs and Equations

In this chapter you will **extend** your understanding of the use of variables in algebra (such as *x* and *y*). You will learn about tools (such as graphing calculators) that will help you explore how variables affect tile patterns, tables, and graphs. You will also continue to develop your ability to solve equations, started in Chapter 2, and will begin a study of the multiple representations of data. You will study the **connections** between graphing and solving equations in Chapter 4.

In this chapter, you will learn:

- How to find a rule from a table.
- How to represent a situation using a table, a rule, and a graph.
- How to graph linear and parabolic rules using an appropriate scale.
- What it means for something to be the solution to an equation and also what it means for an equation to have no solution.
- How to determine the number of solutions to an equation.

Guiding Questions

Think about these questions throughout this chapter:

What is a variable?

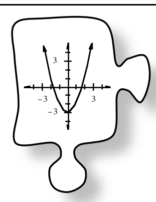
What is the pattern?

How many different ways can I represent it?

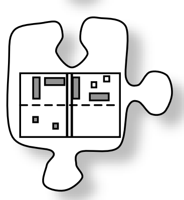
How can I solve it?

How can I check my answer?

Chapter Outline

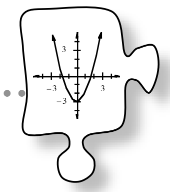


Section 3.1 In this section, you will learn the graphing mechanics that you will need throughout the rest of this course. You will also learn how to create tables, write rules, and draw graphs to represent situations and patterns.



Section 3.2 Section 3.2 will **extend** the work you did in Section 2.2. You will learn how to solve linear equations without using algebra tiles and will learn the significance of solutions.

3.1.1 What is the rule?



Extending Patterns and Finding Rules

You have been learning how to work with variables and how to find values for variables in equations. In this section, you will learn how to **extend** patterns and how to **generalize** your pattern with a rule. As you work with your team, use these questions to focus your ideas:

How is the pattern growing?

What is the rule?

Is there another way to see it?

How can you tell if your rule is correct?

3-1. Some people describe mathematics as "the study of patterns." For each tile pattern below, draw Figure 1 and Figure 5 on graph paper. First try it individually, and then consult with your team. What does Figure 100 look like? Explain how you know.

a.

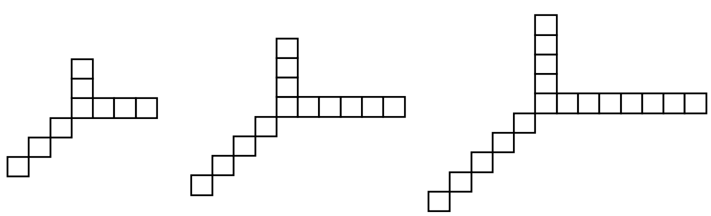


Figure 2 Figure 3 Figure 4

b.

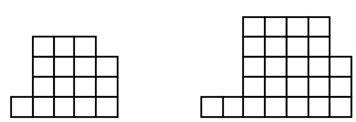


Figure 2 Figure 3 Figure 4

3-2. FINDING RULES FROM TABLES

How can you describe the rule that governs a pattern or table? Obtain the Lesson 3.1.1A Resource Page from your teacher and find the tables below. Then, as a class, find the pattern, fill in the missing parts, and **extend** each table with at least two more $x \rightarrow y$ pairs that fit the pattern. Then **generalize** the pattern's rule in words.

a.

IN (x)	OUT (y)
	C
L	N
	F
Q	
W	Y

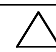
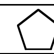
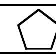
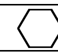
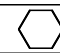
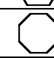
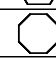
Rule:

b.

IN (x)	OUT (y)
easy	
	light
hot	cold
up	down
left	

Rule:

c.

IN (x)	OUT (y)
	
	
	
	
	

Rule:

d.

IN (x)	OUT (y)
8	17
–2	
	9
12	25
10	21

Rule:

e.

IN (x)	OUT (y)
100	51
4	
6	4
30	16
	31

Rule:

f.

IN (x)	OUT (y)
4	16
–1	1
	9
12	
–6	

Rule:

3-3. Obtain the Lesson 3.1.1B Resource Page from your teacher. For each $x \rightarrow y$ table given, find the pattern and fill in the missing entries. Then write the rule for the pattern in words. Be sure to share your thinking with your teammates.

a.

IN (x)	OUT (y)
	8
0	–2
–4	–10
10	18
–2	
	198
0.5	

Rule:

b.

IN (x)	OUT (y)
3	–9
10	
–1	3
	6
0	
	–36
–5	15

Rule:

c.

IN (x)	OUT (y)
0.5	
	37
2	5
–10	101
–5	
0	1
	50

Rule:

Problem continues on next page →

3-3. *Problem continued from previous page.*

d.

IN (x)	OUT (y)
6	
11	5
	–4
23	17
–7	
	40
–4	–10

Rule:

e.

IN (x)	OUT (y)
2	6
4	20
10	110
–3	
	30
7	56
1	

Rule:

f.

IN (x)	OUT (y)
–8	
10	53
3	18
0	
	8
19	
4	23

Rule:

MATH NOTES

LOOKING DEEPER

Patterns in Nature

Patterns are everywhere, especially in nature. One famous pattern that appears often is called the Fibonacci Sequence, a sequence of numbers that starts 1, 1, 2, 3, 5, 8, 13, 21, …

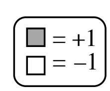
The Fibonacci numbers appear in many different contexts in nature. For example, the number of petals on a flower is often a Fibonacci number, and the number of seeds on a spiral from the center of a sunflower is, too.

To learn more about Fibonacci numbers, search the Internet or check out a book from your local library. The next time you look at a flower, look for Fibonacci numbers!

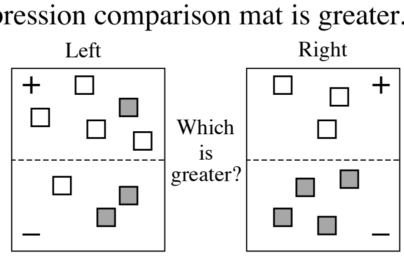
Review & Preview

3-4. WHICH IS GREATER?

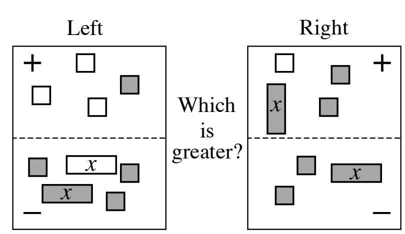
Write the algebraic expressions shown below. Use "legal" simplification moves to determine which expression in the expression comparison mat is greater.



a.



b.



3-5. Evaluate the expressions below for the given values.

a. $3(2x+1)$ for $x=-8$

b. $\frac{x-6}{4}-1$ for $x=-14$

c. $-2m^2+10$ for $m=-6$

d. $k \cdot k \div k \cdot k \div k$ for $k=9$

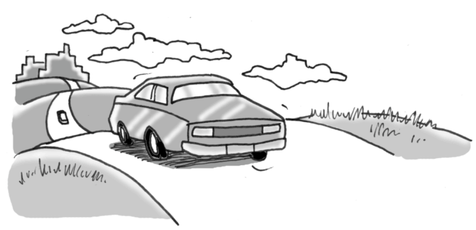
3-6. At the fair, Kate found a strange machine with a sign on it labeled, "Enter a number." When she pushed the number 15, the machine displayed 9. When she entered 23, the machine displayed 17. Perplexed, she tried 100, and the machine displayed 94.

a. What is the machine doing?

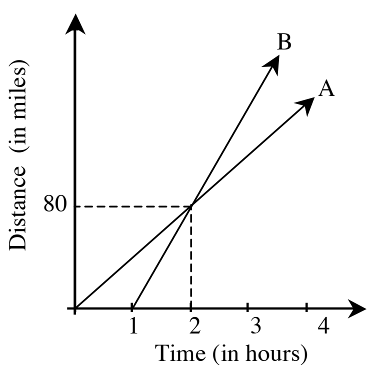
b. What would the machine display if she entered 77?

3-7. Ms. Nguyen needs to separate \$385 into three parts to pay some debts. The second part must be five times as large as the first part. The third part must be \$35 more than the first part. How much money must be in each part?

3-8. GO GOLDEN GOPHERS!



The graph below describes the distance two cars have traveled after leaving a football game at the University of Minnesota.

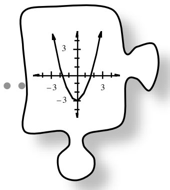


a. Which car was traveling faster? How can you tell?

b. The lines cross at (2, 80). What does this point represent?

c. Assuming that Car A continued to travel at a constant rate, how far did Car A travel in the first 4 hours?

3.1.2 How can I make a prediction?



Using Tables, Graphs, and Rules to Make Predictions

In Lesson 3.1.1, you wrote rules for patterns found in $x \rightarrow y$ tables. In this lesson, you will focus on using variables to write algebraic rules for patterns and contextual situations. You will use a graph to help predict the output for fractional x-values and will then use a rule to predict the output when the input is too large and does not appear on the graph.

While working today, focus on these questions:

How can you write the rule without words?

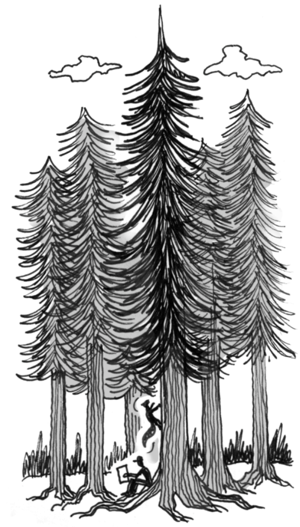
What does x represent?

How can you make a prediction?

3-9. SILENT BOARD GAME

During Lesson 3.1.1, you created written rules for patterns that had no tiles or numbers. You will now write algebraic rules using a table of jumbled in/out numbers. Focus on finding patterns and writing rules as you play the Silent Board Game. Your teacher will put an incomplete $x \rightarrow y$ table on the overhead or board. Study the input and output values and look for a pattern. Then write the rule in words and symbols that finds each y-value from its x-value.

3-10. JOHN'S GIANT REDWOOD, Part One



John found the data in the table below about his favorite redwood tree. He wondered if he could use it to predict the height of the tree at other points of time. Consider this as you analyze the data and answer the questions below. Be ready to share (and **justify**) your answers with the class.

Number of Years after Planting	3	4	5
Height of Tree (in feet)	17	21	25

a. How tall was the tree 2 years after it was planted? What about 7 years after it was planted? How do you know?

b. How tall was the tree the year it was planted?

c. Estimate the height of the tree 50 years after it was planted. How did you make your prediction?

3-11. John decided to find out more about his favorite redwood tree by graphing the data.

a. On the Lesson 3.1.2B Resource Page provided by your teacher, plot the points that represent the height of the tree over time. What does the graph look like?

b. Does it make sense to connect the points? Explain your thinking.

c. According to the graph, what was the height of the tree 1.5 years after it was planted?

d. Can you use your graph to predict the height of the redwood tree 20 years after it was planted? Why or why not?

3-12. John is still not satisfied. He wants to be able to predict the height of the tree at any time after it was planted.

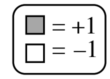
a. Find John's table on your resource page and **extend** it to include the height of the tree in the 0^{th} year, 1^{st} year, 2^{nd} year, and 6^{th} year.

b. If you have not already, use the ideas from the Silent Board Game to write an algebraic rule for the data in your table. Be sure to work with your team and check that the rule works for all of the data.

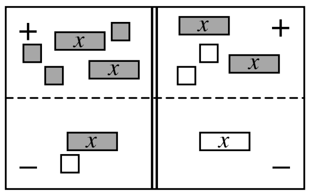
c. Use your rule to check your prediction in part (c) of problem 3-10 for how tall the tree will be in its 50^{th} year. How close was your prediction?

Review & Preview

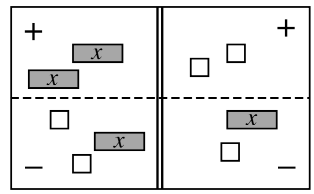
3-13. Write the equation represented in each diagram below on your paper. For each part, simplify as much as possible and then solve for x. Be sure to record your work on your paper.



a.



b.



3-14. Evaluate the following expressions given the values below.

a. $ab + bc + ac$ for $a = 2$, $b = 5$, and $c = 3$

b. $\frac{20-x^2}{y-x}$ for $x = -2$ and $y = 6$

3-15. Copy and simplify the following expressions by combining like terms.

a. $x + 3x - 3 + 2x^2 + 8 - 5x$

b. $2x + 4y^2 - 6y^2 - 9 + 1 - x + 3x$

c. $2x^2 + 30y - 3y^2 + 4xy - 14 - x$

d. $20 + 3xy - 3xy + y^2 + 10 - y^2$

3-16. Use the order of operations to simplify the following expressions.

a. $5 - 2 \cdot 3^2$

b. $(-2)^2$

c. $18 \div 3 \cdot 6$

d. -2^2

e. $(5-3)(5+3)$

f. $24 \cdot \frac{1}{4} \div -2$

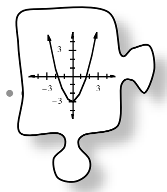
g. Why are your answers for parts (b) and (d) different?

3-17. Mrs. Swanson gives out only one type of candy for Halloween. The local discount store sells six pounds of butterscotch candies for \$7.50. Use proportional reasoning to determine the information below. Be sure to explain your answer and organize your reasoning.

a. What is the cost of 18 pounds of butterscotch candies?

b. What is the cost of 10 pounds of butterscotch candies?

3.1.3 What is a graph and how is it useful?



Using the Graphing Calculator and Identifying Solutions

In the last two lessons, you examined several patterns and learned how to represent the patterns in a table and with a rule. For the next few days, you will learn a powerful new way to represent a pattern and make predictions.

As you work with your team today, use these focus questions to help direct your discussion:

What is the rule?

How can you represent the pattern?

3-18. Find the "Big Cs" pattern shown at right on the Lesson 3.1.3 Resource Page provided by your teacher.

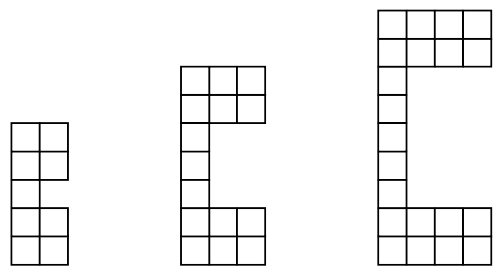


Figure 1 Figure 2 Figure 3

a. Draw Figure 0 and Figure 4 on the grid provided on the resource page.

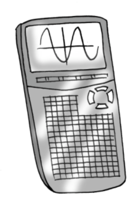
b. On the resource page, represent the number of tiles in each figure with:

- An $x \rightarrow y$ table.
- An algebraic rule.
- A graph.

c. How many tiles will be in Figure 5? **Justify** your answer in at least two different ways.

d. What will Figure 100 look like? How many tiles will it have? How can you be sure?

3-19. Use the graphing technology provided by your teacher to analyze the pattern further and make predictions.



a. Enter the information from your $x \rightarrow y$ table for problem 3-18 into your grapher. Then plot the points using a window of your choice. What do you notice?

b. Find another $x \rightarrow y$ pair that you think belongs in your table. Use your grapher to plot the point. Does it look correct? How can you tell?

c. Imagine that you made up 20 new $x \rightarrow y$ pairs. Where do you think their points would lie if you added them to the graph?

3-20. In the same window that contains the data points, graph the algebraic equation for the pattern from problem 3-18.

a. What do you notice? Why did that happen?

b. Charles wonders about connecting the points of the "Big Cs" data. When the points are connected with an unbroken line or curve, the graph is called **continuous**. If the graph of the tile pattern is continuous, what does that suggest about the tile pattern? Explain.

c. Jessica prefers to keep the graph of the tile-pattern data as separate points. This is called a **discrete** graph. Why might a discrete graph be appropriate for this data?

3-21. If necessary, re-enter your data from the "Big Cs" pattern into your grapher. Re-enter the rule you found in problem 3-18 and graph the data and rule in the same window.

For the following problems, **justify** your conclusions with the *graph*, the *rule*, and the *figure* (whenever practical). Your teacher may ask your team to present your solution to one of these problems. Be sure to justify your ideas using all three representations.

a. Frangelica thinks that Figure 6 in the "Big Cs" pattern has 40 tiles. Decide with your team whether she is correct and **justify** your answer by using the rule, drawing Figure 6, and adding the point to your graph of the data. Be prepared to show these three different ways to **justify** your conclusion.

b. Giovanni thinks that the point (16, 99) belongs in the table for the "Big Cs" pattern. Decide with your team whether he is correct and **justify** your conclusion by examining the graph and the rule.

c. Jeremiah is excited because he has found another rule for the "Big Cs" pattern! He thinks that $y = x + 8$ also works. Prove or disprove Jeremiah's claim. Be prepared to convince the class that your conclusion is correct.

d. LaTanya has been thinking hard and has found another rule for the same pattern! She is sure that $y = 3(2x + 1)$ is also correct. Prove or disprove LaTanya's position in as many ways as you can.

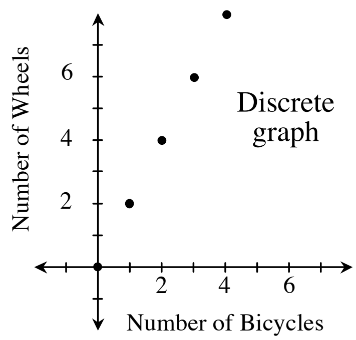
3-22. Look back at the prediction you made in problem 3-18 for Figure 100 in the "Big Cs" pattern. Decide now if your prediction was correct and be ready to defend your position with all of the math tools you have.

MATH NOTES

METHODS AND MEANINGS

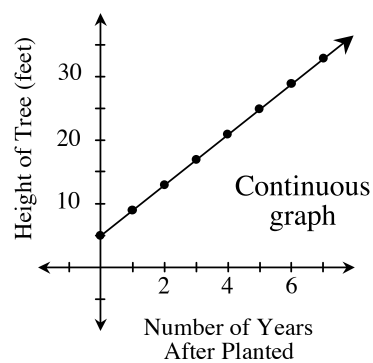
Discrete and Continuous Relationships

When a graph of data is limited to a set of separate, non-connected points, that relationship is called **discrete**. For example, consider the relationship between the number of bicycles parked at your school and the number of bicycle wheels. If there is one bicycle, it has two wheels. Two bicycles have four wheels, while three bicycles have six wheels. However, there cannot be 1.3 or 2.9 bicycles. Therefore, this data is limited because the number of bicycles must be a whole number, such as 0, 1, 2, 3, and so on.



When graphed, a discrete relationship looks like a collection of unconnected points. See the example of a discrete graph above.

When a set of data is not confined to separate points and instead consists of connected points, the data is called **continuous**. "John's Giant Redwood," problem 3-10, is an example of a continuous situation because even though the table focused on integer values of years (1, 2, 3, etc.), the tree still grows between these values of time. Therefore, the tree has a height at any non-negative value of time (such as 1.1 years after it is planted).

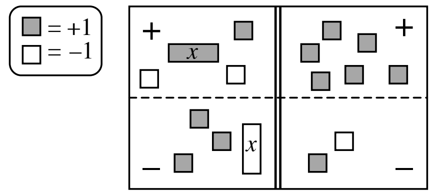


When the data for a continuous relationship is graphed, its points are connected to show that the relationship also holds true for all of the points between the table values. See the example of a continuous graph above.

Note: In this course, tile patterns will represent elements of continuous relationships and will be graphed with a continuous line or curve.

Review & Preview

3-23. On your paper, write the equation represented at right. Simplify as much as possible and then solve for x.



3-24. Find the value of x that makes each equation below true.

a. $x + 7 = 2$

b. $-5 = \frac{1}{2}x$

c. $3x = -45$

d. $2 = -x$

e. $-5 = \frac{x}{2}$

f. $x^2 = 9$ (*all* possible values for x)

3-25. For the following equations, draw a picture of the tiles on an equation mat, simplify, and solve for the variable. Record your work.

a. $3c - 7 = -c + 1$

b. $-2 + 3x = 2x + 6 + x$

3-26. Solve this problem using Guess and Check. Write your solution in a sentence.

West High School's population is 250 students fewer than twice the population of East High School. The two schools have a total of 2858 students. How many students attend East High School?

3-27. Solve the following problems using the order of operations. Show your steps. Verify your answers with your calculator.

a. $(-4)(-2) - 6(2 - 5)$

b. $23 - (17 - 3 \cdot 4)^2 + 6$

c. $14(2 + 3 - 2 \cdot 2) \div (4^2 - 3^2)$

d. $12.7 - 18.5 + 15 + 6.3 - 1 + 28.5$

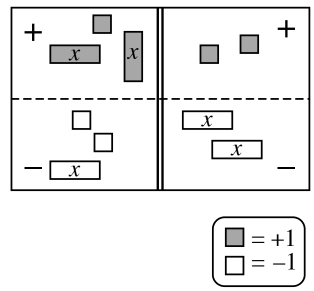
3-28. Copy the table below onto your paper and use your pattern skills to complete it.

IN (x)	2	10	0	7	–3		–10	100	x
OUT (y)	–6	–30	0			15			

a. Explain in words what is done to the input value, x, to produce the output value, y.

b. Write the process you described in part (a) in algebraic symbols.

3-29. Write the equation represented in the equation mat at right.



a. Simplify as much as possible and solve for x.

b. Evaluate both the left side and the right side using your solution from part (a). Remember that if your solution is correct, both sides should have the same value.

3-30. For the following equations, draw a picture of the tiles in an equation mat, use "legal" moves to simplify, and solve for the variable. Record your work.

a. $-3x+7=-x-1$

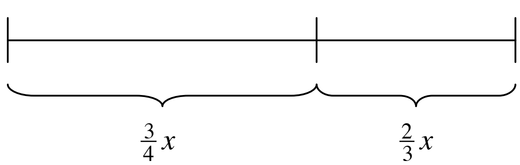
b. $1-2p+5=4p+6$

3-31. Combine like terms in each part below.

a. Liha has three x^2-tiles, two x-tiles, and eight unit tiles, while Makulata has five x^2-tiles and two unit tiles. At the end of class, they put their pieces together to give to Ms. Singh. Write an algebraic expression for each student's tiles and find the sum of their pieces.

b. Simplify the expression $4x+6x^2-11x+2+x^2-19$.

c. Write the length of the line below as a sum. Then combine like terms.



3.1.4 How should I graph?

Completing Tables and Drawing Graphs

In Lesson 3.1.3, you used a graphing tool to represent all of the $x \rightarrow y$ pairs that follow a particular rule. Today you will learn how to make your own graphs for rules and how to recognize patterns that occur in graphs.

3-32. CLASS GRAPH

Your teacher will give your team some x-values. For each x-value, calculate the corresponding y-value that fits the rule $y = -5x + 12$. Then mark the point you have calculated on the class graph.

3-33. Use the rule $y = 2x + 1$ to complete parts (a) through (c) below.

a. Make a table like the one below and use the rule provided above to complete it.

IN (x)	–4	–3	–2	–1	0	1	2	3	4
OUT (y)									

b. Examine the numbers in the table. What are the greatest x- and y-values in the table? What are the smallest x- and y-values? Use this information to set up x- and y-axes that are scaled appropriately.

c. Plot and connect the points on a graph. Be sure to label your axes and write numbers to indicate scale.

3-34. Calculate the y-values for the rule $y = -3x + 1$ and complete the table below.

IN (x)	–4	–3	–2	–1	0	1	2	3	4
OUT (y)									

a. Examine the x- and y-values in the table. Is it possible to use the same set of axes as problem 3-33? If so, graph and connect these points on the axes from problem 3-33. If not, plot and connect the points on a new set of axes.

b. What does your graph look like? Describe the result.

c. How is this graph similar to the graph in problem 3-33? How is it different?

3-35. Calculate the y-values for the rule $y = x^2$ and complete the table below.

IN (x)	–4	–3	–2	–1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4
OUT (y)											

a. Examine the x- and y-values in the table. Use this information to set up a new set of x- and y-axes that are scaled appropriately. Plot and connect the points on your graph, and then label your graph with its rule.

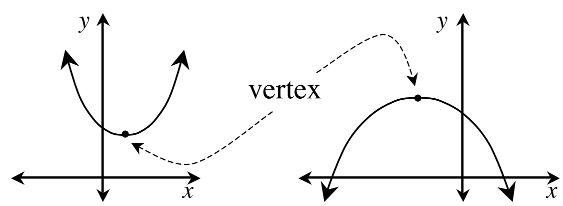
b. This graph is an example of a **parabola**. Read about parabolas in the Math Notes box below. Where is the vertex of the parabola you graphed in part (a)?

METHODS AND MEANINGS

MATH NOTES

Parabolas

One kind of graph you will study in this class is called a **parabola**. Two examples of parabolas are graphed at right. Note that parabolas are smooth "U" shapes, not pointy "V" shapes.



The point where a parabola turns (the highest or lowest point) is called the **vertex**.

Review & Preview

3-36. Complete a table like the one provided in problem 3-33 for the rule $y = x + 2$. Plot and connect the points on graph paper. Be sure to label the axes and include the scale.

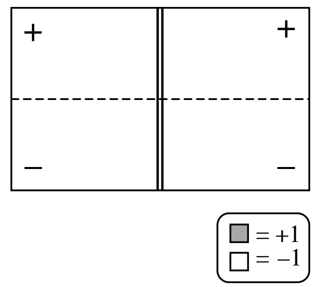
3-37. Use your pattern skills to copy and complete the table below.

IN (x)	2	10	6	7	–3		–10	100	x
OUT (y)	5	21	13			–15			

a. Explain in words what is being done to the input value, x, to produce the output value, y.

b. Write the process you described in part (a) in algebraic symbols.

3-38. For the following equations, draw a picture of the tiles in an equation mat like the one shown at right. Then use "legal" moves to simplify and solve for the variable. Record your work.



a. $-2 + x = -x + 2$

b. $2 + 3x = x + 7$

3-39. Evaluate each expression below.

a. For $y = 5 + 8x$ when $x = 4$, what does y equal?

b. For $a = 3 - 5c$ when $c = -0.5$, what does a equal?

c. For $n = 2d^2 - 5$ when $d = -2$, what does n equal?

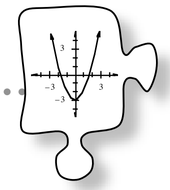
d. For $v = -3(r - 3)$ when $r = -1$, what does v equal?

3-40. Peggy Sue decided to enter her famous "5-Alarm Chili" at her local chili-cooking contest. Normally, she needs five tomatoes to make enough chili for her family of seven.

a. How many tomatoes should she expect to use to make her famous recipe for 100 people?

b. When she gets to the contest, she realizes that she only packed 58 tomatoes. How many people can she expect to feed?

3.1.5 How can I graph it?



Graphs, Tables, and Rules

In Lesson 3.1.4, you practiced setting up the correct axes to graph data from a table. Today you will graph a rule by first making a table, and then by plotting the points from your table on a graph. You will also continue to find patterns in tables and graphs.

3-41. SILENT BOARD GAME

Your teacher will put an incomplete $x \rightarrow y$ table on the board or overhead. Try to find the pattern (rule) that gets each *y*-value from its *x*-value. Find and write the rule for the pattern you find.

3-42. GOOD TIPPER

Mr. Wallis needs your help. He is planning on taking his new girlfriend out to dinner and wants to be prepared to give a tip at the end of the meal. He knows that with any miscalculation, he may leave too little or too much, which might change her view of him and jeopardize their relationship. Therefore, he would like to create a "tip table" that would help him quickly determine how much tip to leave.

a. Create a table like the one shown below. What are reasonable values of *x*? Mr. Wallis needs a tip table that will help him quickly determine a tip for a bill that may occur after a nice dinner for two. Discuss this with your team and then choose eight values for *x*.

Dinner Bill (in dollars)								
Amount of Tip (in dollars)								

b. Mr. Wallis is planning to leave a 15% tip. That means that for a bill of \$10, he would leave a \$1.50 tip. Determine the tip for all of the values in your table from part (a). This is Mr. Wallis's tip table.

c. Use the tip table to estimate the tip quickly if the bill is \$36. What if the bill is \$52.48?

Problem continues on next page →

3-42. *Problem continued from previous page.*

d. Mr. Wallis is worried that he may not be able to quickly estimate using his table for unusual amounts, such as $52.48. He would like a graph to help him determine a 15% tip for *all possible* dollar amounts between $10 and $100. With your team, determine how to set up axes and then graph the points from the tip table. Use the questions below to help guide your discussion.

- Should the tip be graphed on the x-axis or the y-axis? Read the Math Notes box for this lesson about **dependent** and **independent variables** to help you decide.

- Which quadrants are useful for this graph? Why?

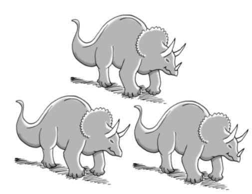
- What are the greatest and smallest values of x and y that must fit on the graph? How can you scale your axes to create the most effective graph for Mr. Wallis?

e. Use your tip graph from part (d) to test your estimations in part (c). Which representation (table or graph) helped to find the most accurate tip? Which was easiest to use? Explain.

3-43. ONE OF THESE POINTS IS NOT LIKE THE OTHERS

a. Plot and connect the points in the table below.

IN (x)	–2	4	1	6	–5	0
OUT (y)	–6	–2	–3	2	–9	–4



b. Identify the point that does not appear to fit the pattern.

c. Correct the point found in part (b) above so it fits the pattern.

3-44. Copy and complete the table below for the rule $y = \frac{1}{2}x + 6$.

IN (x)	–4	–3	–2	–1	0	1	2	3	4
OUT (y)									

a. Graph and connect the points from your table on graph paper. Remember to label the graph with its rule.

b. Does the point (10, 12) lie on this graph? How can you tell?

MATH NOTES

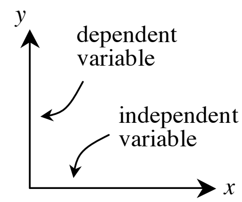
METHODS AND MEANINGS

Independent and Dependent Variables

When one quantity (such as the height of a redwood tree) depends on another (such as the number of years after the tree was planted), it is called a **dependent variable**. That means its value is determined by the value of another variable. The dependent variable is usually graphed on the y-axis.

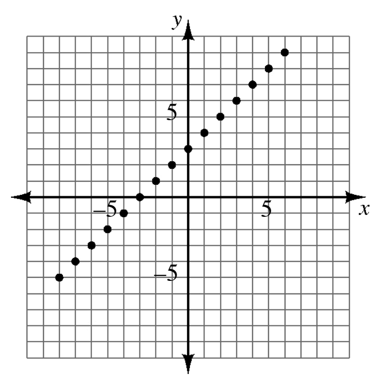
If a quantity, such as time, does not depend on another variable, it is referred to as the **independent variable**, which is graphed on the x-axis.

For example, in problem 3-42, you compared the amount of a dinner bill with the amount of a tip. In this case, the tip depends on the amount of the dinner bill. Therefore, the tip is the dependent variable, while the dinner bill is the independent variable.



Review & Preview

3-45. Create an $x \rightarrow y$ table using at least eight points from the graph at right. Write the rule for the pattern in the table.



3-46. For each rule below, make a table of x- and y-values. Then graph and connect the points from your table on graph paper using an appropriate scale. Label each graph with its equation.

a. $y = -2x + 7$

b. $y = 11x$

3-47. On graph paper, draw Figure 0 and Figure 4 for the pattern at right.

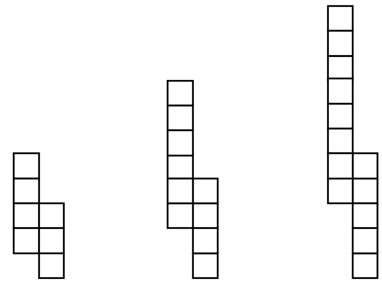


Figure 1 Figure 2 Figure 3

a. Represent the number of tiles in each figure in an $x \to y$ table. Let x be the figure number and y be the total number of tiles.

b. Use the table to graph the pattern.

c. Without drawing Figure 5, predict where its point would lie on the graph. **Justify** your prediction.

3-48. Use your pattern skills to copy and complete the table below.

IN (x)	2	10	6	7	–3		–10	100	x
OUT (y)	2	6	4			15			

a. Explain in words what is done to the input value, x, to produce the output value, y.

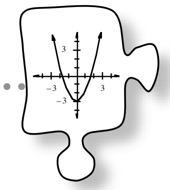
b. Write the process you described in part (a) in algebraic symbols.

3-49. For the following equations, simplify and solve for the variable. Record your work.

a. $2x - 7 = -2x + 1$

b. $-2x - 5 = -4x + 2$

3.1.6 What makes a complete graph?



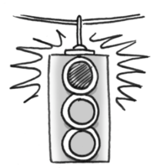
Complete Graphs

Over the past several days you have learned to make graphs from tables, then graphs from rules. Today you will continue to study graphs by deciding what needs to go into a graph to make it complete.

3-50. SILENT BOARD GAME

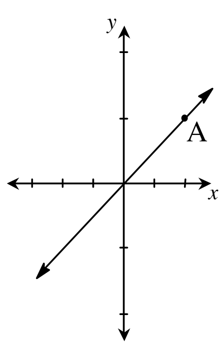
Your teacher will put an incomplete $x \rightarrow y$ table on the board or overhead. Try to find the pattern (rule) that gets each y-value from its x-value. Find and write the rule for the pattern you find.

3-51. Examine the following graphs and answer the question associated with each one. What do you notice?

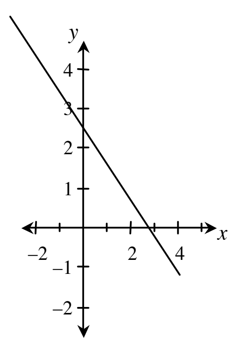


Note: This stoplight icon will appear periodically throughout the text. Problems with this icon display common errors that can be made. Be sure not to make the same mistakes yourself!

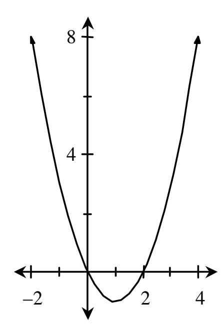
a. What are the coordinates of point A?



b. Where will the line be when $x = 5$?



c. What is t when $k = 1$?



3-52. On your own graph paper, graph $y = -3x + 2$. Then, as a class, decide what needs to be included to make a graph complete. Copy the qualities of a complete graph as a Learning Log entry. Title this entry “Qualities of a Complete Graph” and include today’s date.

3-53. Make your own complete graph for each of the following rules:

a. $y = -x + 1$ b. $y = 0.5x + 2$ c. $y = x^2 - 4$

3-54. Examine the graphs from problem 3-53.

a. How are they different? Be as specific as you can.

b. Label the (x, y) coordinates on each of your graphs for the point where each graph crosses the y-axis. These points are called ***y*-intercepts**.

c. Label the (x, y) coordinates on each of your graphs for the point or points where each graph crosses the x-axis. These points are called ***x*-intercepts**.

Review & Preview

3-55. Complete a table for the rule $y = x^2 + 2$. Then plot and connect the points on a graph. Be sure to label the axes and include the scale. Use negative, positive, and zero values for x.

3-56. Complete a table for the rule $y = -x + 3$. Then plot and connect the points on a graph. Be sure to label the axes and include the scale. Use negative, positive, and zero values for x.

3-57. On graph paper, draw Figure 0 and Figure 4 for the pattern below. Describe Figure 100 in detail.

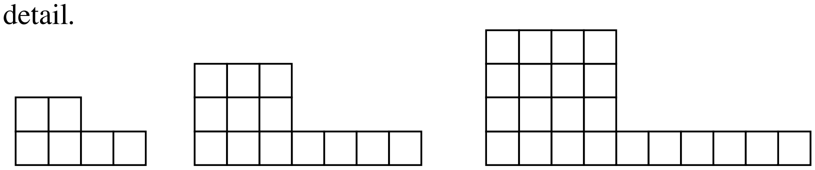
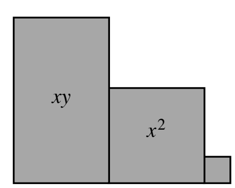


Figure 1 Figure 2 Figure 3

3-58. Write an expression that represents the perimeter of the shape built with algebra tiles at right. Then find the perimeter if $x = 3$ units and $y = 7$ units.

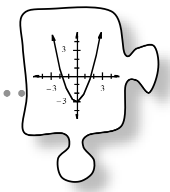


3-59. For the following equations, draw a picture of the tiles on an equation mat, use "legal" moves to simplify, and then solve for the variable. Record your work.

a. $3x - 7 + 3 + 2x = -x + 2$ b. $-2k + 5 + (-k) + 1 = 0$

3.1.7 What is wrong with this graph?

Identifying Common Graphing Errors



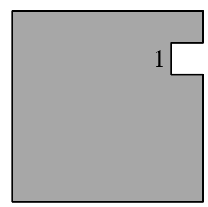
In this chapter you have used rules to find y-values to go with x-values in tables. Then you graphed the $x \rightarrow y$ pairs you found. Today you will be examining how rules, tables, and graphs can be used to represent new situations. You will also learn how to avoid common graphing errors. As you work, revisit the following questions:

What x-values should go in my table?

How can I correct this error?

How should I scale my graph?

3-60. Ms. Cai's class is studying the "dented square" shape shown at right. This shape is formed by removing a square with side length 1 from a larger square. Her students decided to let x represent the side length of the large square and y represent the perimeter of the entire shape.



a. What is the perimeter of the "dented square?" That is, what rule could help you find the perimeter for any value of x?

b. Make a table for the rule Ms. Cai's class found. Make sure the x-values you use are appropriate for this situation. What are the possible x-values?

c. Make a graph from your $x \rightarrow y$ table.

d. Do you think the points on your graph should be connected? **Justify** your answer.

3-61. GOOFY GRAPHING

x	*y*
2	8
3	17
5	35
6	44

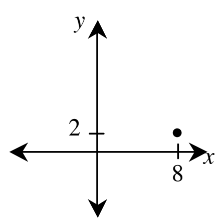
Now Ms. Cai's class is studying a tile pattern. Her students decided to represent the pattern with the $x \rightarrow y$ table at right.

a. Ms. Cai wants her class to graph the data in this table. Write (x, y) coordinates for each point that needs to be plotted.

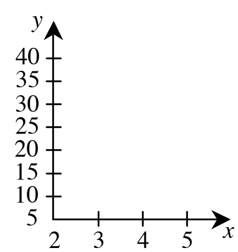
b. When Ms. Cai's students started to graph this data, they made mistakes right from the beginning. The diagrams below show how some of Ms. Cai's students set up their axes. Your teacher will assign your team one of these diagrams.

Your Task: Find all of the mistakes the students made in setting up the graph your teacher assigns you. (There may be more than one mistake in each graph!) Explain why this is an incorrect way to set up a graph, or why this is not the best way to set up the graph for this problem. Be ready to present your team's ideas to the class.

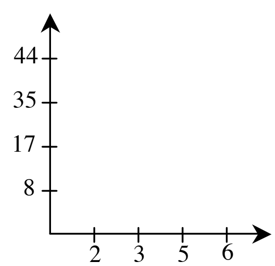
i.



ii.



iii.



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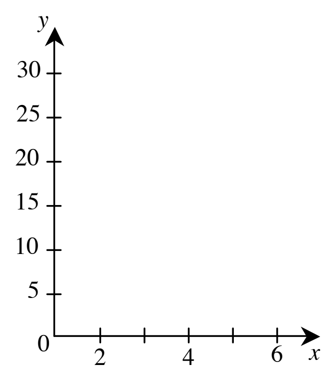
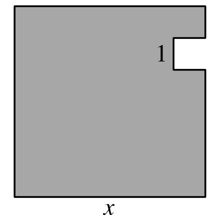


3-62. Sheila is in Ms. Cai's class. She noticed that the graph of the perimeter for the "dented square" in problem 3-60 was a line. "I wonder what the graph of its area looks like," she said to her teammates.



a. Write an equation for the area of the "dented square" if x represents the length of the large square and y represents the area of the square.

b. On graph paper, graph the rule you found for the area in part (a). Why does a 1^{st}-quadrant graph make sense for this situation?

c. Explain to Sheila what the graph of the area looks like.

d. Use the graph to approximate x when the area of the shape is 20 square units.

3-63. Looking back at the mistakes Ms. Cai's students made, write a Learning Log entry that includes a checklist of errors you should make sure to avoid when setting up a graph. Title this entry "Graphing Errors" and label it with today's date.

Review & Preview

3-64. Pat delivers newspapers every morning. On average, she can deliver newspapers to 17 homes in 12 minutes.

a. If her normal route has 119 homes, how long does it take her to deliver the mail on average?

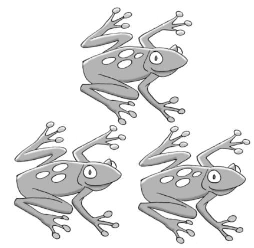
b. When another deliverer is sick, Pat's route expands to a total of 155 homes. How long does this route take her?

3-65. ONE OF THESE POINTS IS NOT LIKE THE OTHERS, Part Two

a. Plot and connect the points listed in the table below.

IN (x)	–2	4	1	–4	0	3	–3	2
OUT (y)	0	12	–3	12	–4	5	–2	0

b. Identify the point that does not fit the pattern.



c. What shape does the graph appear to make?

d. Correct the point identified in part (b) so it fits the pattern. Write the points in (x, y) notation.

3-66. For each rule below, make a table of x- and y-values and then graph the rule on graph paper. Label each graph with its equation.

a. $y = x^2$

b. $y = -x^2$

c. Compare the graphs. What do you notice?

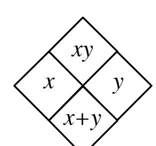
d. For the graph of $y = x^2$, estimate the x-values corresponding to $y = 5$.

e. For the graph of $y = -x^2$, estimate the x-values corresponding to $y = -10$.

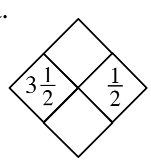
3-67. Paris is trying to solve the equation $3x^2 - (2x - 4) = 3 + 3x^2 + 1$. Her work is partially recorded below. Copy her table and fill in her missing work to solve for x.

Left Expression	**Right Expression**	**Explanation**
$3x^2 - (2x - 4)$	$3 + 3x^2 + 1$	Starting expressions.
$3x^2 - 2x + 4$	$3 + 3x^2 + 1$	
		Remove $3x^2$ from both sides.
$-2x$	0	
		Divide both sides by –2.

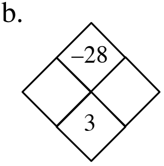
3-68. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



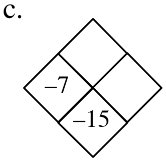
a.



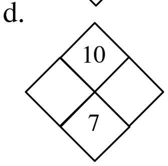
b.



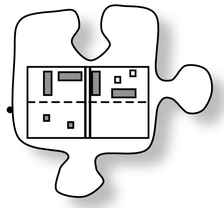
c.



d.



3.2.1 How can I check my answer?



Solving Equations and Testing the Solution

In Section 2.2, you learned to solve equations on an equation mat. In this section, you will practice your equation-solving skills while adding a new element: You will check your answer to make sure it is correct.

While solving equations in this lesson, keep these focus questions in mind:

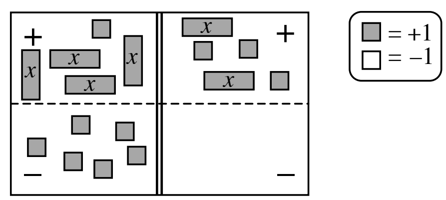
What is your goal?

How can you start?

How can you simplify?

Can you get x alone?

3-69. For this activity, share algebra tiles and an equation mat with your partner.



a. Start by setting up your equation mat as shown at right. Write the equation on your paper.

b. Next, solve the equation on your equation mat one step at a time. Every time you make a step, record your work in two ways:

- Record the step that was taken to get from the old equation to the new equation.
- Write a new equation that represents the tiles in the equation mat.

c. With your partner, find a way to check if your solution is correct.

3-70. WHAT IS A SOLUTION?

In this lesson you have found solutions to several algebraic equations. But what exactly is a solution? Answer each of these questions with your study team, but *do not use algebra tiles*. Be prepared to **justify** your answers!

a. Preston solved the equation $3x-2=8$ and got the solution $x=100$. Is he correct? How do you know?

b. Edwin solved the equation $2x+3-x=3x-5$ and got the solution $x=4$. Is he correct? How do you know?

c. With your partner, discuss what you think a solution to an equation is. Write down a description of what you and your partner agree on.

3-71. Work with your partner to solve these equations, being careful to record your work. After solving each equation, be sure to check your solution, if possible.

a. $3x + 4 = x + 8$ b. $4 - 2y = y + 10$

c. $5x + 4 - 2x = -(x + 8)$ d. $-2 - 3k - 2 = -2k + 8 - k$

3-72. IS THERE ANOTHER WAY?

Compare your solution to part (c) of problem 3-71 with the solution that another pair of students got. Did both solutions involve the same steps? Were the steps used in the same order? If not, copy the other pair's solution onto your paper. If both pairs used the same steps in the same order to solve the equation, come up with a different way to solve the problem and record it on your paper.

METHODS AND MEANINGS

MATH NOTES

Complete Graph

A complete graph has the following components:

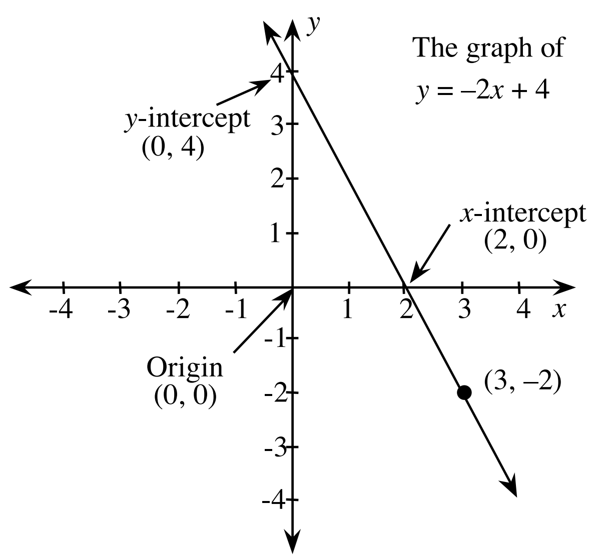
- x-axis and y-axis labeled, clearly showing the scale.
- Equation of the graph near the line or curve.
- Line or curve extended as far as possible on the graph.
- Coordinates of special points stated in (x, y) format.

x	-1	0	1	2	3
y	6	4	2	0	-2

Tables can be formatted horizontally, like the one above, or vertically, as shown below.

x	y
-1	6
0	4
1	2
2	0
3	-2
4	-4

Throughout this course, you will continue to graph lines and other curves. Be sure to label your graphs appropriately.



Review & Preview

3-73. For the following equations, solve for x. Be sure to check your answer, if possible. Show all work.

a. $3x + 7 = -x - 1$

b. $1 - 2x - 5 = 4x + 2$

c. $-3x = x - (6 - 2x)$

d. $2x + 3 = -2x + 5$

3-74. For the rule $y = -2x + 1$, calculate the y-values that complete the table below.

IN (x)	–4	–3	–2	–1	0	1	2	3	4
OUT (y)									

a. Graph your rule on a set of axes. Be sure to create a complete graph. If necessary, see the Math Notes box for this lesson to review what makes a graph complete.

b. Describe your resulting graph. What does your graph look like?

3-75. Simplify each expression below.

a. $4x + 7 + 3y - (1 + 3y + 2x)$

b. $16x^2 - 4x + 5 - (16x^2 - 8x) + 1$

c. $(32x - 7y) - (28x - 11y)$

d. $y + 2 + 2y + 2 + 2y - 2x + y$

3-76. Burgers-o-rama is the best hamburger place in town. The owner, Ms. Hamm, buys two 5-pound packages of meat for \$27.50. Use proportional reasoning to determine the information below. Be sure to explain your answer and organize your reasoning.

a. What should Ms. Hamm pay for 25 pounds of meat?

b. How many pounds can Ms. Hamm buy for \$55.00?

3-77. I'm thinking of a number. When I double my number, subtract five, and then add my original number, I get one. What's my number?

3.2.2 How many solutions are there?

Determining the Number of Solutions

In Lesson 3.2.1, you reviewed your equation-solving skills to remember how to find a solution to an equation. But do all equations have a solution? And how can you tell if an equation does not have a solution?

Today you will continue to practice solving equations and will continue to investigate the meaning of a solution.

3-78. GUESS MY NUMBER

Today you will play the "Guess My Number" game. You will need a pencil and a piece of paper. Your teacher will think of a number and tell you some information about that number. You will try to figure out what your teacher's number is. (You can use your paper if it helps.) When you think you know what the mystery number is, sit silently and do not tell anyone else. This will give others a chance to think about it.

3-79. Use the process your teacher illustrated to analyze Game #3 of "Guess My Number" algebraically.

a. Start by writing an equation that expresses the information in the game.

b. Solve your equation, writing down each step as you go. When you reach a conclusion, discuss how it agrees with the answer for Game #3 you found as a class.

c. Repeat this process to analyze Game #4 algebraically.

3-80. How many solutions does each equation below have? To answer this question, solve these equations, recording all of your steps as you go along. Check your solution, if possible.

a. $4x-5=x-5+3x$

b. $-x-4x-7=-2x+5$

c. $3+5x-4-7x=2x-4x+1$

d. $4x-(-3x+2)=7x-2$

e. $x+3+x+3=-(x+4)+(3x-2)$

f. $x-5-(2-x)=-3$

3-81. In your Learning Log, explain how to find the number of solutions to an equation. How do you know when an equation has no solution? How do you know when an equation has an infinite number of solutions? Give examples of each kind of equation, as well as an equation with exactly one solution. Title your entry "How Many Solutions?" and label it with today's date.

Review & Preview

3-82. Create your own "Guess My Number" game like the ones you worked with in class today. Start it with, "I'm thinking of a number that…" Make sure it is a game you know the answer to! Write the equation and solve it.

3-83. Draw Figure 0 and Figure 4 for the pattern below on graph paper.

a. Represent the number of tiles in each figure with:

- An $x \to y$ table.
- An algebraic rule.
- A graph.

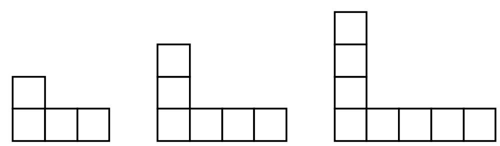


Figure 1 Figure 2 Figure 3

b. Without drawing Figure 5, predict where its point would lie on the graph. **Justify** your prediction.

3-84. For the following equations, simplify and solve for the variable. Show all work and check your solution, if possible.

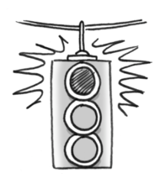
a. $-2 + x = -x + 2$

b. $-(x - 1) = -4x - 2$

c. $2 + 3x = 3x + 2$

d. $-(-x + 6) = -3x$

3-85. For each equation, a possible solution is given. Check to see if the given solution is correct.



a. $3x + 7 = x - 1$, solution $x = -4$?

b. $-2x - 4 = -4x + 3$, solution $x = 3$?

c. $-3x + 5 + 5x - 1 = 0$, solution $x = 2$?

d. $-(x - 1) = 4x - 5 - 3x$, solution $x = 3$?

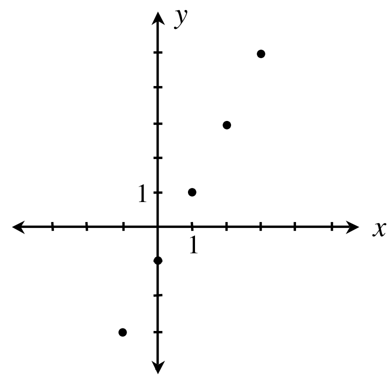
3-86. Examine the graph at right.

a. Use the graph to complete the table:

IN (x)					
OUT (y)					

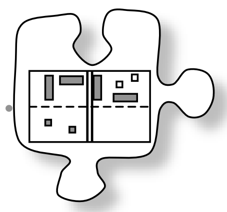
b. Use the graph to find the rule:

$y =$ _________



3.2.3 How can I use my equation-solving skills?

Solving Equations to Solve Problems



In the last two lessons you have practiced solving equations. In this lesson you will **apply** your equation-solving skills to the patterns you found at the beginning of this chapter. As you solve these problems, keep these questions in mind:

How can you simplify?

Is there more than one way to solve?

Can you get x alone?

How can you check your solution?

3-87. In Lesson 3.1.3, you investigated the "Big Cs" pattern of tiles, shown at right. The rule you found for this pattern was $y = 6x + 3$, where x represented the figure number and y represented the number of tiles in the figure.

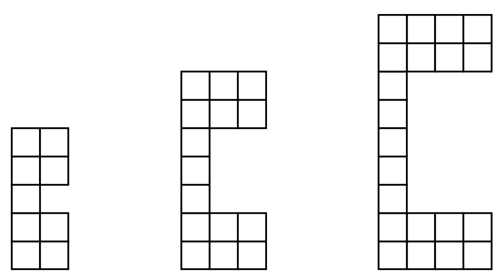


Figure 1 Figure 2 Figure 3

Penelope wants to know how many tiles will be in Figure 50. How can you determine this? Write out in words what you would need to do with your rule to answer her question. Then answer Penelope's question: How many tiles will be in Figure 50?

3-88. Lew wants to **reverse** the process: He says he has a "Big Cs" figure made up of 45 tiles. He wants to know which figure number this pattern is.

a. In the rule $y = 6x + 3$, which variable must equal 45 to solve Lew's problem?

b. Write an equation you could use to solve Lew's problem. Then solve your equation, recording all of your steps. Which "Big Cs" figure is made up of 45 tiles?

c. How can you check your answer to be sure it is correct? Check your solution.

3-89. Norm says he has a "Big Cs" pattern made up of 84 tiles. He wants to know which figure number this pattern is. Write and solve an equation as you did in problem 3-88. Does your solution make sense? Why or why not?

3-90. For the following equations, solve for x. Record your work and check your solution.

a. $\frac{1}{2}x - 2 = x - 4$

b. $8 - 0.25x = 0.5x + 2$

c. $x + 2 - 0.5x = 1 + 0.5x + 1$

d. $7x - 0.15 = 2x + 0.6$

3-91. Can an equation be solved using a graph? Consider this as you answer the questions below.

a. Solve the equation $5 = 1.6x + 1$. Check your solution.

b. Complete a table for the rule $y = 1.6x + 1$. Then, on graph paper, graph the line.

c. Use the graph from part (b) to find x when $y = 5$. Did you get the same result as in part (a)?

Review & Preview

3-92. Evaluate the expressions below for the given values.

a. $30-2x$ for $x=-6$

b. x^2+2x for $x=-3$

c. $-\frac{1}{2}x+9$ for $x=-6$

d. $\sqrt{k}$ for $k=9$

3-93. For the following equations, solve for x. Check your solution, if possible. Record your work.

a. $3x-7=3x+1$

b. $-2x-5=-4x+2$

c. $2+3x=x+2+2x$

d. $-(x-2)=x+2$

3-94. The length of a rectangle is three centimeters more than twice the width. The perimeter is 78 centimeters. Use Guess and Check to find out how long and how wide the rectangle is.

3-95. Use a diagram of the equation mat or some other method to explain why $-(x-3)=-x+3$.

3-96. For the rule $y=4-x^2$, calculate the y-values that complete the table below. The first value is given for you.

IN (x)	–3	–2	–1	0	1	2	3
OUT (y)	–5						

a. Create an x-axis and a y-axis and label your units. Plot and connect the points on your graph, and then label your graph with its rule.

b. What does your graph look like?

3.2.4 How can I use my equation-solving skills?

More Solving Equations to Solve Problems

3-97. JOHN'S GIANT REDWOOD, Part Two

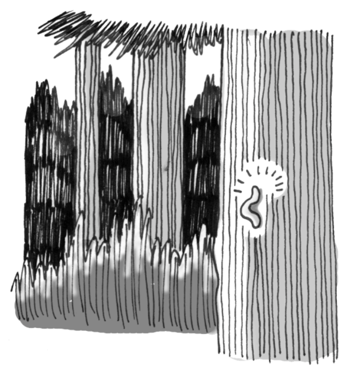
In Lesson 3.1.2, you looked at how a tree increases in height as it gets older. Review the data below and, if possible, find your work from problem 3-10.

Number of Years after Planting	3	4	5
Height of Tree (in feet)	17	21	25

a. Assuming the tree continues to grow at a constant rate, find a rule for the height of the tree using x and y.

b. In your rule, what real-world quantity does x stand for? What real-world quantity does y stand for?

c. John wants to know how tall the tree will be when it is 20 years after planting. Use your rule to answer his question.



d. The tallest tree in the world, in Montgomery Woods State Reserve in California, is 367 feet high. John wants to know how long it would take for his tree to get that tall if it keeps growing at the same rate. Write and solve an equation you could use to answer John's question. Be sure to check your solution.

e. Did you use algebra tiles to solve the equation in part (d)? Would it be easy to use algebra tiles to do so? Why or why not?

3-98. For the following equations, solve for the given variable. Record your work and check the solution, if possible.

a. $75c - 300 = 25c + 200$

b. $26y - 4 - 11y = 15y + 6$

c. $-\frac{1}{2}x = 6$

d. $0.8 - 2t = 1 - 3t$

3-99. MR. WALLIS IS BACK!

After much consideration, Mr. Wallis decided to use the tip table below to help him estimate what a 15% tip would be for various costs of dinner.

Cost of Dinner	$10	$20	$30	$35	$40	$45	$50	$100
Amount of Tip	$1.50	$3	$4.50	$5.25	$6	$6.75	$7.50	$15

a. Find a rule for his table. That is, find a rule that calculates the amount of tip (y) based on the cost of the dinner (x). How did you find your rule?

b. During the date, Mr. Wallis was so distracted that he forgot to write down the cost of the meal in his checkbook. All he remembers is that he left a $9 tip. What was the original cost of the meal before he paid the tip? Use your equation from part (a) to answer this question. Show all work.

c. What was the total cost of the meal?

METHODS AND MEANINGS

MATH NOTES

Solutions to an Equation with One Variable

A **solution** to an equation gives a value of the variable that makes the equation true. For example, when 5 is substituted for x in the equation at right, both sides of the equation are equal. So $x = 5$ is a solution to this equation.

$$4x - 2 = 3x + 3$$
$$4(5) - 2 = 3(5) + 3$$
$$18 = 18$$

An equation can have more than one solution, or it may have no solution. Consider the examples at right.

Equation with no solution:
$$x + 2 = x + 6$$

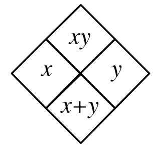
Equation with infinite solutions:
$$x - 3 = x - 3$$

Notice that no matter what the value of x is, the left side of the first equation will never equal the right side. Therefore, we say that $x + 2 = x + 6$ has **no solution**.

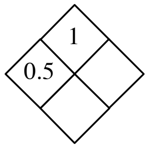
However, in the equation $x - 3 = x - 3$, no matter what value x has, the equation will always be true. So all numbers can make $x - 3 = x - 3$ true. Therefore, we say the solution for the equation $x - 3 = x - 3$ is **all numbers**.

Review & Preview

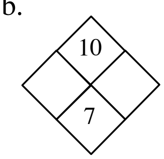
3-100. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



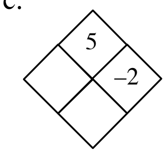
a.



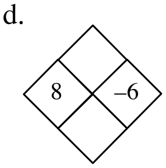
b.



c.



d.



3-101. The science club is selling homemade cookies to raise money for a field trip. They know that 12 dozen cookies uses 3 pounds of flour. Use proportional reasoning to determine the information below. Be sure to explain your answer and organize your reasoning.

a. How much flour is needed for 18 dozen cookies?

b. How many cookies can be made with 10 pounds of flour?

3-102. Simplify each of the following equations and solve for the variable. Show all work and check your solution, if possible.

a. $3x - 7 + 9 - 2x = x + 2$ b. $-2m + 8 + m + 1 = 0$

c. $2 = x + 6 - 2x$ d. $0.5p = p + 5$

3-103. Use your pattern-finding skills to copy and complete the table below.

IN (x)	1	2	3		5	6	8	12	24	x
OUT (y)	24	12		6	4.8	4		2		

a. Explain the pattern you found in your table. How are x and y related?

b. Write the rule you described in part (a) in algebraic symbols.

c. Use the points in your table to graph this rule on graph paper. Describe the resulting shape.

3-104. Translate each algebraic expression into ordinary words.

a. $5x - 3$ b. $2(x + y)$ c. $3 - (x + 5)$

Chapter 3 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

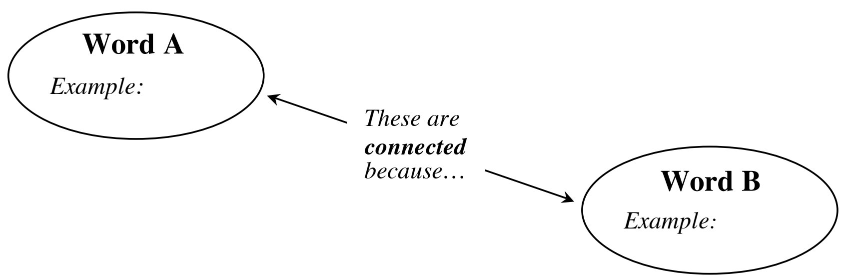
Connections: What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

area	**continuous**	coordinates
dependent variable	**discrete**	equation
equation mat	evaluate	**figure number**
graph	**independent variable**	**input value (x)**
output value (y)	**parabola**	pattern
prediction	quadrant	**rule**
scale on axes	simplify	solution
variable	**$x \rightarrow y$ table**	**x- and y-intercept**

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch that illustrates the idea (see the example below).



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this. Your teacher may give you a "GO" page to work on. The "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section appears at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try the problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 3-105. For the $x \to y$ table below, fill in the missing values and find the rule.

IN (x)	–10	0	5	1	25	–6	8	–1	6	10
OUT (y)		3		5	53	–9				23

CL 3-106. One year ago, Josie moved into a new house and noticed a beautiful vine growing on the back fence. She recorded the data in the table below.

Weeks Since Josie Moved In	4	5	6
Height of Vine (in inches)	16	19	22

Assuming that the vine continues to grow at a constant rate:

a. How tall was the vine 7 weeks after Josie moved in?

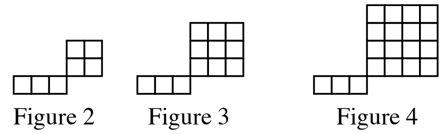
b. How tall was the vine 3 weeks after Josie moved in?

c. How tall was the vine when Josie moved in? How do you know?

d. Predict how tall the vine was 19 weeks after Josie moved into her house. **Justify** your answer.

e. Predict when the vine reached the top of the garage (94 inches tall). How did you find your answer?

CL 3-107. Examine the tile pattern at right.



a. Draw Figure 1 and Figure 5.

b. Make an $x \to y$ table for the pattern.

c. Make a complete graph. Include points for Figures 0 through 5.

CL 3-108. Clifford is making a cake for his sister's birthday. The recipe calls for twice as much flour as sugar. It also calls for 20 ounces of ingredients other than flour and sugar. All the ingredients together total 80 ounces. How much flour does Clifford need?

CL 3-109. Simplify the expression $3x^2 - 5x - 4 + xy - (2xy + 2x^2)$. Then evaluate the result if $x = -1$ and $y = 6$.

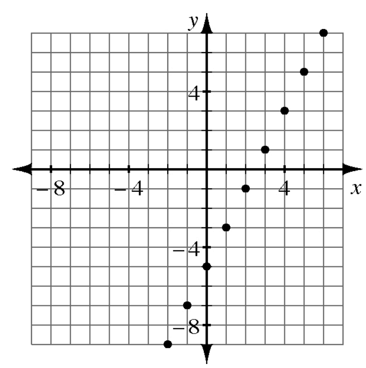
CL 3-110. Follow the order of operations to simplify.

a. $6^2 - (5 - 4) + 2(8 - 2^2) \div 8$

b. $\frac{2(9-6)^2}{18}$

CL 3-111. Raphael had 5 hits in 7 at bats. If he continues this pattern, how many hits will he have in 210 at bats?

CL 3-112. Solve $6 - x - 3 = 4x - 12$ for x, recording your steps as you work.



CL 3-113. Make an $x \to y$ table from the points on the graph at right. Then write a rule for the table.

CL 3-114. Make an $x \to y$ table and complete graph for the equation $y = -2x + 5$.

CL 3-115. Jessica was solving an equation. After she finished simplifying, her result was $0 = 2$. This result confused her. Explain to Jessica what her result means. Explain your reasoning thoroughly.

CL 3-116. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

This closure activity will focus on one of these Ways of Thinking: **generalizing**. Read the description of this Way of Thinking at right.

Generalizing

To generalize means to make a general statement or conclusion about something from partial evidence. You think this way when you describe patterns, because you are looking for a general statement that describes each term in the pattern. Often, a generalization is the answer to the question, "What is in common?" When you catch yourself thinking, "*I think this is always true…*", you are generalizing.

Think about the topics that you have learned during this chapter. When did you need to describe patterns? When did you draw a conclusion or make a **general** statement? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any ideas you have with the rest of the class.

Once your discussion is complete, examine some of the ways you have **generalized** as you answer the questions below.

a. Examine the tile pattern below.

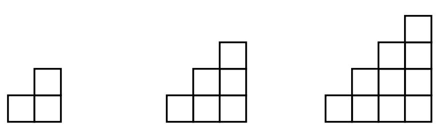


Figure 1 Figure 2 Figure 3

i. Draw Figure 4 and Figure 5 on graph paper.

ii. How do the figures appear to be growing? Make a **general** statement describing how all of the figures change to become the next figure in the pattern.

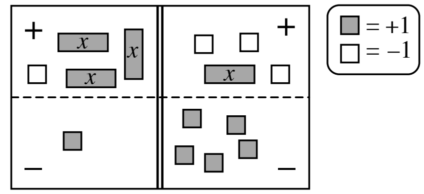
iii. Sketch and describe Figure 100.

iv. In **general**, what does each figure look like? That is, describe Figure n.

Continues on next page →

⑤ *Continued from previous page.*

b. During Section 3.2, you probably **generalized** about how to solve equations. For example, in Chapter 2, you solved equations with algebra tiles, such as those shown on the equation mat at right. However, during this chapter, you have solved some equations that cannot easily be expressed using algebra tiles. How? Because you **generalized** the process to include any type of linear equation, such as $8-0.25x=0.5x+2$ from problem 3-90. Consider this as you answer the questions below.



i. In **general**, what are some strategies you can use no matter how many *x*-tiles and unit tiles are placed on both sides of the equation?

ii. Use the general strategies you described in (*i*) above to solve the equation below. Show all work and check your solution.

$$\tfrac{1}{4}x-(3-\tfrac{3}{4}x)=\tfrac{1}{2}x-7$$

iii. Some equations have no solution. In **general**, describe how you know if an equation has no solution. Because you do not have a specific equation to solve, this description must help to describe all situations in which an equation has no solution. This is another example of a **generalization**.

c. Examine the data in the table below. Find a rule that describes how all of the *x*-values and *y*-values are related. Since this rule describes a **general** property that all of these points share, it is also an example of a **generalization**.

IN (x)	–10	0	5	1	25	–6	8	–1	6	10
OUT (y)	–26	4	19	7	79	–14	28	1	22	34

Answers and Support for Closure Activity #4
What Have I Learned?

<table>
<tr><th>Problem</th><th>Solution</th><th>Need Help?</th><th>More Practice</th></tr>
<tr><td>CL 3-105.</td><td>The missing y-values, in order, are: –17, 13, 19, 1, 15. The rule is $y = 2x + 3$.</td><td>Lesson 3.1.1</td><td>Problems 3-2, 3-3, 3-28, 3-37, 3-48, and 3-103</td></tr>
<tr><td>CL 3-106.</td><td>a. 25 inches
b. 13 inches
c. 4 inches
d. 61 inches
e. 30 weeks after Josie moved in</td><td>Lesson 3.1.2</td><td>Problems 3-10 and 3-97</td></tr>
<tr><td>CL 3-107.</td><td>a.

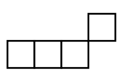



Figure 1

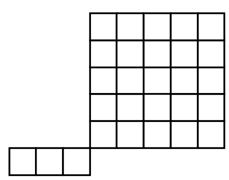
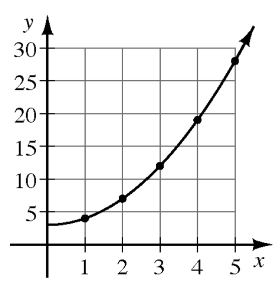
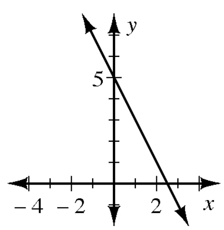



Figure 5

b.
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>4</td><td>7</td><td>12</td><td>19</td><td>28</td></tr>
</table>

c.


Note: In this course, tile patterns will represent elements of continuous relationships and will be graphed with a continuous line or curve.</td><td>Section 3.1</td><td>Problems 3-1, 3-18, 3-47, 3-57, and 3-83</td></tr>
</table>

<table>
<tr><th>Problem</th><th>Solution</th><th>Need Help?</th><th>More Practice</th></tr>
<tr><td>CL 3-108.</td><td>40 ounces</td><td>Lesson 2.1.7 Math Notes box</td><td>Problems 3-7, 3-26, 3-77, and 3-94</td></tr>
<tr><td>CL 3-109.</td><td>$x^2 - xy - 5x - 4$
$= (-1)^2 - (-1)(6) - 5(-1) - 4$
$= 8$</td><td>Lessons 2.1.1, 2.1.3, 2.1.5, and 2.1.6 Math Notes boxes</td><td>Problems 3-5, 3-14, 3-15, 3-31, 3-39, 3-75, and 3-92</td></tr>
<tr><td>CL 3-110.</td><td>a. 36
b. 1</td><td>Lesson 2.1.3 Math Notes box</td><td>Problems 3-16 and 3-27</td></tr>
<tr><td>CL 3-111.</td><td>150 hits</td><td>Lessons 2.2.1 and 2.2.3</td><td>Problems 3-17, 3-40, 3-64, 3-76, and 3-101</td></tr>
<tr><td>CL 3-112.</td><td>$x = 3$</td><td>Lesson 2.1.8 Math Notes box</td><td>Problems 3-24, 3-25, 3-30, 3-38, 3-49, 3-59, 3-73, 3-84, and 3-90</td></tr>
<tr><td>CL 3-113.</td><td><table><tr><td>x</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>–9</td><td>–7</td><td>–5</td><td>–3</td><td>–1</td><td>1</td><td>3</td><td>5</td><td>7</td></tr></table>$y = 2x - 5$</td><td>Lesson 3.1.4</td><td>Problems 3-45 and 3-86</td></tr>
<tr><td>CL 3-114.</td><td><table><tr><td>x</td><td>–3</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>11</td><td>9</td><td>7</td><td>5</td><td>3</td><td>1</td><td>–1</td><td>–3</td></tr></table></td><td>Lesson 3.1.4, Lesson 3.2.1 Math Notes box</td><td>Problems 3-46, 3-55, 3-56, 3-66, 3-74, and 3-96</td></tr>
<tr><td>CL 3-115.</td><td>There is no solution for x in this equation.</td><td>Lesson 3.2.1, Lesson 3.2.4 Math Notes box</td><td>Problems 3-71, 3-80, and 3-93</td></tr>
</table>